Comprehensive Comparison of Medical Image Reconstruction Algorithms: Application to Positron Emission Mammography

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Abstract
Image reconstruction is the image processing technique such that it creates two or three-dimensional images from radiation scattering information in one dimension with different angles of view. It has a key role in discovering inside knowledge of part of human body without further operation such as surgery or endoscopy. Therefore, the usage and popularity of imaging methods which are based on image reconstruction increase substantially. Image reconstruction algorithms have an importance because clear and meaningful images are very helpful for doctors being able to diagnose diseases easily. Image reconstruction algorithms are split into two parts such as analytical and iterative algorithms. Analytical methods are based on direct mathematical foundations and they require many samples to reconstruct better image. Moreover, they are independent from imaging system structure. Hence it is easy to apply each system. However, iterative algorithms need system information and generate better image with quite less samples. In this study, general image reconstruction algorithms are investigated and compared with each other in respect of image quality measures and these algorithms' applicability to positron emission mammography are discussed.

Keywords: Reconstruction of PET, medical imaging, image reconstruction methods

1. INTRODUCTION

Image reconstruction is a process which is generating two or three dimensional images by using gathered radiation scattering data in one dimension with different angles of view. Reconstructed image is so important that it gives in vivo information for experts. Due to the fact that tomographic images do not require any surgical operation, it is very popular the usage and popularity of imaging methods which are based on image reconstruction increase substantially [1].
Medical imaging techniques which uses image reconstruction are divided into two categories in aspect of radiation source. Computer tomography (CT) is a typification for radiation source located out of body. Radiation source and sensors are located in straight angle and they rotate around the part of body which is preferred to observe [2]. CT is used when anatomical information is necessary. On the other hand, one of the systems which radiation source is placed in body is positron emission tomography (PET) [3]. In PET imaging, radioactive traced pharmaceutics are injected into tissue and sensors are located around this part. Therefore, PET is a functional imaging technique which has applications in oncology, cardiology and neurology, e.g. for monitoring tumours or visualizing coronary artery disease. In other words; it is used when physiological information is in need.
Image reconstruction is very meaningful for doctors since it provides clear and meaningful images that are helpful for doctors being able to diagnose diseases easily.
Image reconstruction algorithms are split into two groups: analytical and iterative algorithms [4]. Analytical methods are based on direct mathematical foundations the implementations are easy however they need a large number of samples to reconstruct better image. However,

iterative algorithms generate better image with quite less samples but they have also some handicaps like needing system information.

In this study, the most popular image reconstruction algorithms are investigated and compared with each other in respect of mean square error and these algorithms' applicability to PEM are discussed according to results.

2. METHODS

2.1 Image Reconstruction Methods

The set of collected line of response (LORs) which account for each coincidence event is the only information provided by PET data. This data is organized into 2D data which is the visual representation of Radon transform and is called as a sinogram which is the most common way to use in respect to angular and radial samplings. In order to obtain an image of radiotracer's distribution, the sinogram must be projected back. There are two basic methods to reconstruct image such as analytical and iterative methods.

2.1.1 Analytical Methods

Analytical methods are based on direct mathematical foundations which are non-realistic and simplified. In addition, these methods are generally formulated for continuous measurements and arranged by integral solutions. Analytical approaches necessitate standard geometries such as complete sampling in angular coordinates.

The foundation of analytical reconstruction methods is Radon transform that is a transformation of a function $f(x,y)$ which is defined by the distance ρ from the origin and the angle of slope θ was introduced by the Austrian mathematician Johann Radon in 1917 [5]. Radon transform of a function $f(x,y)$ can be expressed with $R_\theta(\rho)$.

$$R_\theta(\rho) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(xcos\theta + ysin\theta - \rho)dxdy \tag{1}$$

where $\delta(.)$ is the Dirac delta function.

Simple Back propagation Algorithm

Back projection method is the simplest reconstruction method which is blurry due to the fact that $f(x,y)$ is reconstructed by sum of line integrals which is duplicated. Backprojection method $b(x,y)$, is

$$b(x,y) = \int_0^{\pi} R_\theta(\rho)d\theta \tag{2}$$

where $\rho = xcos\theta + ysin\theta$, that shows the summation of $R_\theta(xcos\theta + ysin\theta)$for all θ. If we substituting equation of Radon transform, in Equation 2 and after simplification we get,

$$= f(x,y) * \left[\frac{1}{\sqrt{x^2 + y^2}}\right] \tag{3}$$

where * represents the convolution operator.

Simple back projection has blurry result because of the factor, $\frac{1}{\sqrt{x^2+y^2}}$ so it is not meaningful, some methods are developed to decrease the effect of blur such as back filtered projection and filtered back projection.

2.1.2 Iterative Methods

Drawback of analytical methods for image reconstruction is that system information is not taken into account. In iterative reconstruction, system information represents as system matrix, A, which includes attenuation and any linear blurring mechanisms.
The general concepts of iterative reconstruction are shown in figure. The process begins with an initial estimate of the pixel values of image. Initial estimate is generally simple such as uniform image. In the next step, estimated image is transformed into projection that this step is called forward projection and can be expressed as

$$p_i = \sum_j a_{ij} f_j \tag{4}$$

In a similar manner, back projection can be expressed as

$$f_j = \sum_i a_{ij} p_i \tag{5}$$

It is summing up the intensities along ray paths for all angles through the estimated image and all projections form into sinogram. Estimated sinogram is compared with actual sinogram, and the difference is used to adjust estimated image to achieve closer approximation. The process is repeated until the difference between estimated image and actual image is small enough.
There are two basic factors for iterative algorithms such as comparison part of estimated and actual image and update part of estimated image according to comparison. In general, comparison part is implemented by the cost function and update part is implemented by using the search function. A general purpose of iterative algorithms is to produce converged the estimated image as speedy and accurately as possible by usage of these functions.

The iterative algorithms are listed in Table 1.

Table 1: Iterative Image Reconstruction Techniques

Method	Expression
Algebraic Reconstruction Technique (ART) [6]	$f_j^{k+1} = f_j^k + \lambda \left[\frac{p_i - \sum_{j=1}^{N} f_j^k a_{ij}}{\sum_{j=1}^{N} a_{ij}^2} \right] a_{ij}$ $for\ all\ i = 1,2, \ldots\ldots, M$
Multiplicative Algebraic Reconstruction Technique (MART) [7]	$f_j^{k+1} = f_j^k \left(\frac{p_i}{\sum_{j=1}^{N} a_{ij} f_j^k} \right)^{a_{ij}}$
Simultaneous Algebraic Reconstruction Technique (SART) [8]	$f_j^{k+1} = f_j^k + \lambda \frac{\sum_i \frac{p_i - \sum_{j=1}^{N} f_j^k a_{ij}}{\sum_{j=1}^{N} a_{ij}} a_{ij}}{\sum_{i=1}^{M} a_{ij}}$
Maximum-Likelihood Expectation-Maximization (MLEM) [9]	$f_j^{k+1} = \frac{f_j^k}{\sum_{i=1}^{M} a_{ij}} \sum_{i=1}^{M} \frac{p_i}{\sum_{j=1}^{N} a_{ij} f_j^k} a_{ij}$
Image Space Reconstruction Algorithm (ISRA) [10]	$f_j^{k+1} = f_j^k \frac{\sum_{i=1}^{M} a_{ij} p_i}{\sum_{i=1}^{M} a_{ij} \sum_{j=1}^{N} a_{ij} f_j^k}$
Weighted Least Square (WLS) [11]	$f_j^{k+1} = f_j^k \sum_{i=1}^{M} \frac{a_{ij} p_i^2}{\left(\sum_{j=1}^{N} a_{ij} f_j^k \right)^2}$

where p_i is measured projection data, k is the iteration index and λ is a relaxation factor from (0,1].

2.2 Photon Noise

An essential sort of light measurement uncertainty and the independence of photon detections is called photon noise. It is also known as Poisson noise. Individual photon detections follow a random temporal distribution which is because they are independent events. Thus, photon counting is clarified by Poisson process that describes the number of photons N measured by a sensor element over a time interval t as

$$P(N = c) = \frac{e^{-\lambda t} (\lambda t)^k}{k!} \quad (6)$$

where λ is the expected number of photons per unit time interval. Due to the fact that photon counting follows a Poisson distribution, it comes by the properties of Poisson distribution that its expectation is equal to its variance. Consequently, it is signal dependent and its standard deviation increases according to the square root of the signal.
In this study, photon noise is modelled by three different λ parameters such as 10^2, 10^4 and 10^6. Effects of using different λ parameters are shown on Derenzo phantom in figure.

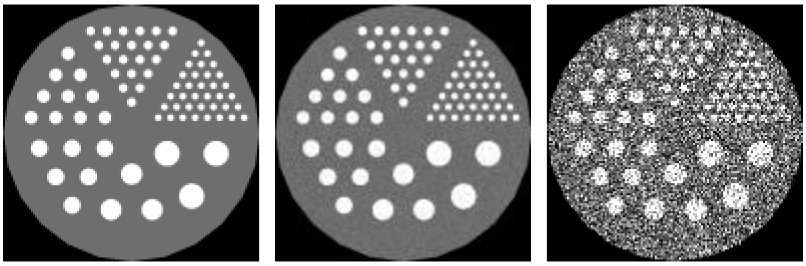

Figure 1. Derenzo phantoms with $\lambda = 10^6$ (left), $\lambda = 10^4$ (center) and $\lambda = 10^2$ (right)

3. RESULTS

Normalized mean square error (MSE) is used as error metric which is described with

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(f_{ij} - f'_{ij})^2. \tag{7}$$

Filtered back projection method is improved version of simple back projection. Projection data is filtered before back projection procedure begins. There are two common filter used in simulations such as Ram-Lak and Sheep-Logan filters. The mean square errors and constructed images are listed below.

Table 2: The mean square errors of filtered back projection algorithm with different filters and noise levels.

Noise Level / Filter Type	$\lambda = 10^2$	$\lambda = 10^4$	$\lambda = 10^6$
Ram-Lak Filter	0.4757	0.4247	0.4302
Sheep-Logan Filter	0.4463	0.4252	0.4272

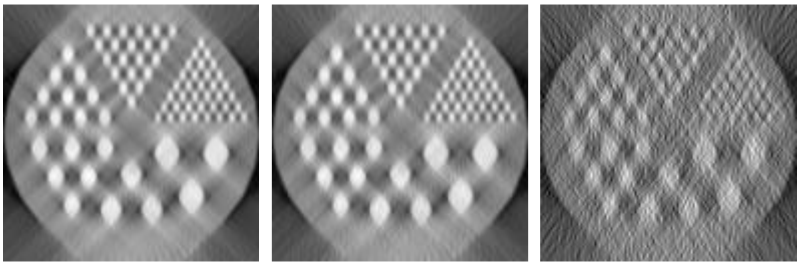

Figure 2. Reconstructed images using Ram-Lak filter for [-45,45] degrees with $\lambda = 10^6$ (left), $\lambda = 10^4$ (center) and $\lambda = 10^2$ (right)

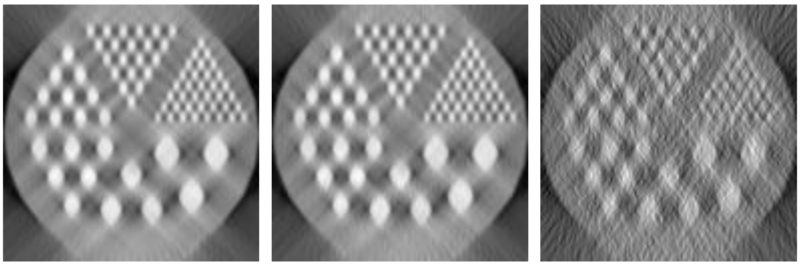

Figure 3. Reconstructed images using Shepp-Logan filter for [-45,45] degrees with $\lambda = 10^6$ (left), $\lambda = 10^4$ (center) and $\lambda = 10^2$ (right)

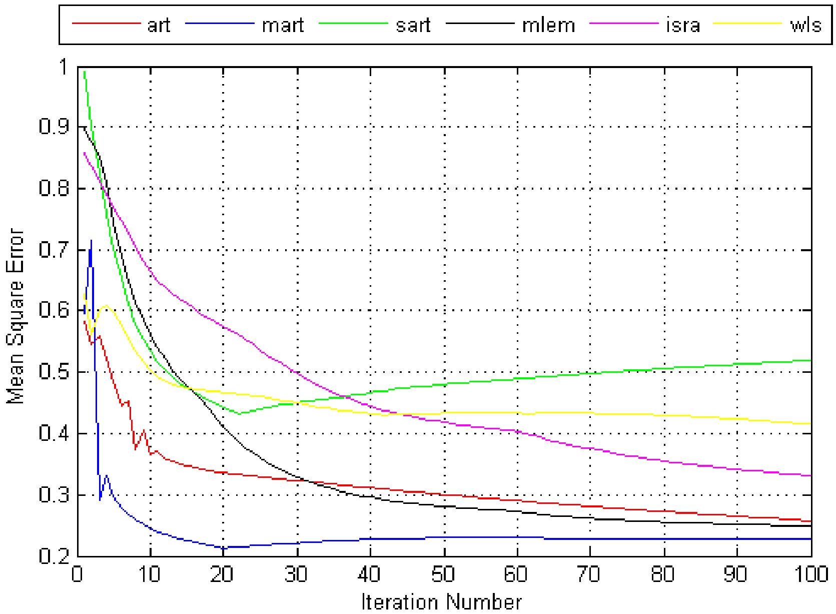

Figure 4. Mean Square Error for Iterative Images Reconstruction Algorithms for $\lambda = 10^6$

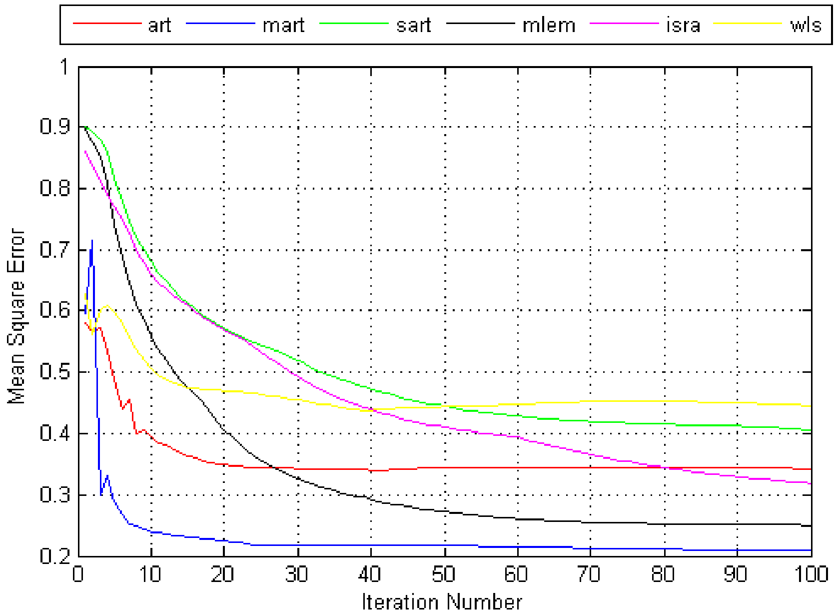

Figure 5. Mean Square Error for Iterative Images Reconstruction Algorithms degrees with $\lambda = 10^4$

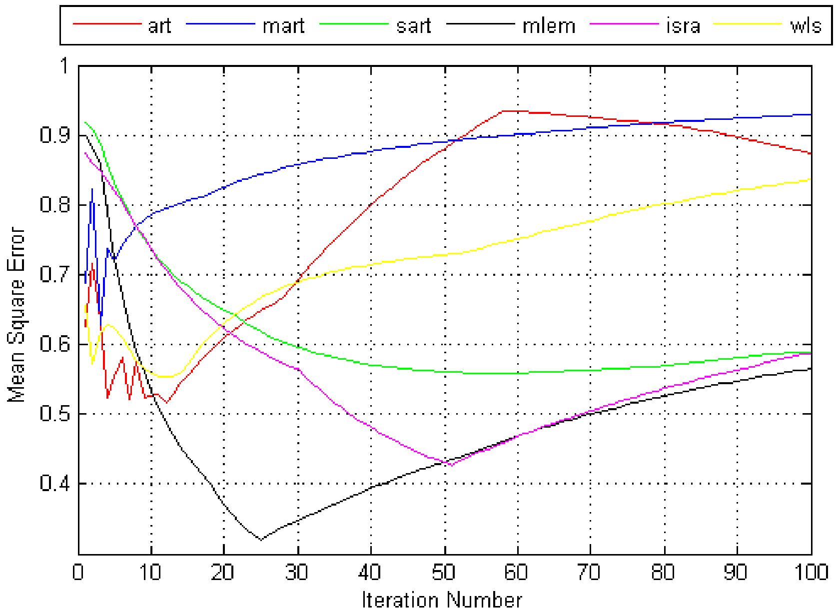

Figure 6. Mean Square Error for Iterative Images Reconstruction Algorithms for $\lambda = 10^2$

4. CONCLUSIONS & RESULTS

We have shown reconstructed images and their output of image quality measurements for both analytical and iterative algorithms. We have also pointed out how different photon counts influence on performance of reconstruction algorithms.

In analytical algorithms, SBP algorithm has the worst results in respect to MSE and reconstructed images as expected because it suffers from blur factor which is explained in Methods section.

FBP algorithm has reduced blur factor by using two different filters such as Ram-Lak and Shepp-Logan.

In iterative algorithms, algebraic methods have reached their best results with less iteration then statistical methods in high photon count. We can say that there is no supremacy between algebraic and statistical methods it depends on the noise level. Statistical methods give the best result when photon count is dramatically decreased while iterative algrebraic methods gives better results for the images who has high signal-to-noise ratio.

5. REFERENCES

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